Simulating and analyzing order book data: The queue-reactive model

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Outline

1. Summary of this work
2. Dynamics of the LOB: constant reference price
3. Dynamic reference price and time consistent model
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Summary

Aim of this work

- Understanding the behaviours of market participants at different limits of the order book.
- Providing a realistic market simulator, enabling to compute execution costs of complex trading strategies.

Approach

- State (order book) dependent order flow intensities, in contrast to the normal Poisson approach.
- Empirical validation through full order book data analysis.
Modelling ideas

- Very intricate dependences between different limits.
- We first model the dynamics of the order book for periods where a 'reference price' stays constant.
- Then we introduce a dynamic behaviour for the reference price.
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Limit order book in this framework

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General framework

Limit order book as a continuous time Markov jump process

- The 2K dimensional LOB state $X(t)$:
  \[ X(t) = (Q_{-K}(t), \ldots, Q_{-1}(t), Q_1(t), \ldots, Q_K(t)) \]
- Order flow intensities: $\lambda^M_{buy/sell}$ (market orders), $\lambda^L_i$ (limit orders at $Q_i$) and $\lambda^C_i$ (cancellations at $Q_i$)
- The associated infinitesimal generator matrix $Q_{x,y}$

\[
\begin{align*}
Q_{x,x+e_i} &= \lambda^L_i(x) \\
Q_{x,x-e_i} &= \lambda^C_i(x) + \lambda^M_{buy}(x)1_{\text{best} \text{bid}(x)=i}, \text{ if } i > 0 \\
Q_{x,x-e_i} &= \lambda^C_i(x) + \lambda^M_{sell}(x)1_{\text{best} \text{ask}(x)=i}, \text{ if } i < 0 \\
Q_{x,x} &= -\sum_{y \in \Omega, y \neq x} Q_{x,y} \\
Q_{x,y} &= 0, \text{ otherwise, (1)}
\end{align*}
\]
Three different models in this framework

<table>
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<tr>
<th>Models and assumptions</th>
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<td><strong>Model I</strong> : Collection of independent queues.</td>
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<td>- Order arrival intensities at $Q_i$ are functions of the queue size.</td>
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<td>- Dynamics at different queues are independent.</td>
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<td><strong>Model II</strong> : Two sets of dependent queues.</td>
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<td>- The information set is enlarged to include the notion of 'best limit'.</td>
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<td>- Price priority is correctly modelled.</td>
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<td><strong>Model III</strong> : Modelling bid-ask dependence.</td>
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<td>- Dependences between the bid and ask queues are added to the model.</td>
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<td>- Ergodicity conditions and stationary laws of $X(t)$ are studied.</td>
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<td>- Tools : Meyn &amp; Tweedie, Quasi.birth-and-death process ...</td>
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Intensities as functions of the target queue size: first limits

Model I, first limit, limit order insertion intensity

Model I, first limit, limit order cancellation intensity

Model I, first limit, market order consumption intensity

Model I, first limit, order arrival/departure ratio
Intensities as functions of the target queue size: second limits

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The queue-reactive model
Invariant Distribution vs. empirical distribution

At Distance 0.5 tick

At Distance 2.5 ticks

At Distance 1.5 ticks
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Purely order book driven model

Dynamics of $p_{\text{ref}}$ in the purely order book driven model

- Changes of $p_{\text{ref}}$ are triggered by changes of the mid price, with probability $\theta$.
- $Q_i$ becomes either $Q_{i+1}$ or $Q_{i-1}$ when $p_{\text{ref}}$ changes.

Problems of this model

- Strong mean-reverting behaviours of the price process.
- The maximum achievable volatility (mechanical volatility) is often smaller than its empirical value.
The queue-reactive model

Dynamics of $p_{\text{ref}}$ in the queue-reactive model

- Changes of $p_{\text{ref}}$ are triggered by changes of the mid price, with probability $\theta$.
- $Q_i$ becomes either $Q_{i+1}$ or $Q_{i-1}$ when $p_{\text{ref}}$ changes.
- The whole LOB is redrawn from its invariant distribution with probability $\theta^{\text{reinit}}$ around the new $p_{\text{ref}}$ when $p_{\text{ref}}$ changes.

Calibration of $\theta$ and $\theta^{\text{reinit}}$

- 10-minute price volatility of the asset.
- Oscillation ratio $\eta$ of the mid price.
A simulated bid-ask trajectory

The queue-reactive model
An application example: comparing two different order placement strategies